STAGE di PALAZZOLO (BS) - Giorno 10

Titolo nota 01/03/2019

0	1	2	3	4	5	6	7	8	9
2	5	8	11	14	17	20	23	26	29
a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9 ...

$$\boxed{a_n = a_{n-1} + 3 , \quad a_0 = 2 \quad \forall n \geq 1} \Longrightarrow a_n = 2 + 3 \cdot n$$

Succ Aritmetica

Succ. Geometrica: $\boxed{a_n = 3a_{n-1}, \quad a_0 = 7 \quad \forall n \geq 1} \Longrightarrow a_n = 7 \cdot 3^n$

$$7, \quad 3 \cdot 7, \quad 3 \cdot 3 \cdot 7, \quad \ldots$$

Es: $a_n = 2 \cdot a_{n-1} - 1 , \quad a_0 = 7$

$$b_n = a_n + k \quad \forall n \geq 0$$

$$\boxed{a_n = b_n - k}$$

$$b_n - k = 2(b_{n-1} - k) - 1$$

$$b_n = 2b_{n-1} - k - 1$$ Se $k = -1$, allora $b_n = 2b_{n-1}$

$\Rightarrow b_n = a_n - 1$, allora $\boxed{b_n = 2b_{n-1}, \quad b_0 = 6}$

$$b_n = 2^n \cdot 6$$

$$\Rightarrow a_n = b_n - k = b_n + 1 = 2^n \cdot 6 + 1$$

$\Rightarrow a_n = 2^n \cdot 6 + 1 \leftarrow$ num. di monete dopo n giorni.

Es:

1) $a_n = 3a_{n-1} - 2 \qquad a_0 = 2 \longrightarrow a_n = 3^n + 1$

2) $a_n = \frac{1}{2} a_{n-1} + \frac{1}{3} \qquad a_0 = 1 \longrightarrow a_n = \frac{1}{3}\left(\frac{1}{2}\right)^n + \frac{2}{3}$

3) $a_n = -2a_{n-1} + 1 \qquad a_1 = 0$

4) $a_n = 2a_{n-1} + n \qquad a_0 = 1$

3) $a_n = -2a_{n-1} + 1, \quad a_1 = 0$

$b_n = a_n + k \qquad a_n = b_n - k$

$b_n - k = -2(b_{n-1} - k) + 1, \quad b_1 = 0 - k = -k$

$b_n = -2b_{n-1} \boxed{+2k+1+k}$

$k = -\frac{1}{3}$

$b_n = -2b_{n-1}$

$b_1 = \frac{1}{3}$

e non n!!!

$b_n = \frac{1}{3}(-2)^{\boxed{n-1}}$

$a_n = b_n - k = \frac{1}{3}(-2)^{n-1} + \frac{1}{3}$

4) $a_n = 2a_{n-1} + n \qquad a_0 = 1$

0	1	2	3	4	5	6	7	8
1	3	8	19	42	89	...	...	...

provo: $b_n = a_n + k \qquad a_n = b_n - k$

$b_n - k = 2(b_{n-1} - k) + n$

$b_n = 2b_{n-1} - 2k + k + n$

$b_n = 2b_{n-1} - k + n$ NON VA

provo: $b_n = a_n + k \cdot n + h$

$a_n = b_n - k \cdot n - h$

$$a_n = 2(a_{n-1}) + n$$

$$b_n - k\cdot n - h = 2\left(b_{n-1} - k(n-1) - h\right) + n$$

$$b_n = 2b_{n-1} - 2k(n-1) - 2h + kn + h + n$$

$$-2kn + 2k - 2h + kn + h + n = 0$$

$$n(k+1-2k) + 2k - h = 0$$

$$n(1-k) + 2k - h = 0$$

$$\Rightarrow \begin{cases} 1-k=0 \\ 2k-h=0 \end{cases} \Rightarrow \begin{cases} k=1 \\ h=2 \end{cases}$$

$$\Rightarrow b_n = a_n + n + 2 \qquad a_n = b_n - n - 2$$

$$b_n = 2b_{n-1}, \quad b_0 = 1 + 0 + 2 = 3$$

$$b_n = 2^n \cdot 3$$

$$\Downarrow$$

$$a_n = 2^n \cdot 3 - n - 2$$

Es: 5) $a_n = 3a_{n-1} - n - 1, \quad a_0 = 5$

6) $a_n = \frac{1}{2}a_{n-1} + 2n - 3, \quad a_0 = 2$

7) $a_n = 2a_{n-1} + n^2, \quad a_0 = 1$

8) $a_n = -2a_{n-1} + 2^n, \quad a_0 = 1$

6) $a_n = \frac{1}{2}a_{n-1} + 2n - 3 \qquad b_n = a_n + kn + h$

$a_0 = 2$

$$b_n - kn - h = \frac{1}{2}\left(b_{n-1} - k(n-1) - h\right) + 2n - 3$$

$$b_n = \frac{1}{2}b_{n-1} - \frac{1}{2}kn + \frac{1}{2}k - \frac{1}{2}h + kn + h + 2n - 3$$

$$n\left(-\frac{1}{2}k+k+2\right)+\left(\frac{1}{2}k-\frac{1}{2}h+h-3\right)$$

$$\begin{cases} \frac{1}{2}k+2=0 & k=-4 \\ \frac{1}{2}k+\frac{1}{2}h=3 & h=10 \end{cases}$$

$$b_n=\frac{1}{2}b_{n-1}$$

$$b_0=a_0-4\cdot 0+10 = 2+10=12$$

$$b_n=12\cdot\frac{1}{2^n} \qquad a_n=12\cdot\frac{1}{2^n}+4n-10$$

7) $a_n=2a_{n-1}+n^2$, $a_0=1$

$$b_n=a_n+k\cdot n^2+h\cdot n+l$$

$$\left(b_n-kn^2-hn-l\right)=2\left(a_{n-1}-k(n-1)^2-h(n-1)-l\right)+n^2$$

$$k=1 \quad h=4 \quad l=6$$

$$a_n=b_n-n^2-4n-6 \qquad b_n=7\cdot 2^n$$

$$b_0=a_0+0+4\cdot 0+6=7$$

$$a_n=7\cdot 2^n-n^2-4n-6$$

8) $a_n=-2a_{n-1}+2^n$, $a_0=1$

$$b_n=a_n+c\cdot 2^n$$

$$a_n=b_n-c\cdot 2^n$$

$$b_n-c\cdot 2^n=-2\left(b_{n-1}-c\cdot 2^{n-1}\right)+2^n$$

$$b_n=-2b_{n-1}+c2^n+c2^n+2^n$$

$$b_n=-2b_{n-1}+2^n(2c+1) \qquad c=-\frac{1}{2}$$

$$b_0 = 1 - \frac{1}{2} \cdot 2^0 = \frac{1}{2} \qquad b_n = \frac{1}{2}(-2)^n$$

$$a_n = \frac{1}{2}(-2)^n + \frac{1}{2} 2^n = \frac{(-2)^n + (2)^n}{2}$$

Oss: $2^{n+k} = 2^k \cdot 2^n = c \cdot 2^n \qquad k = \log_2 c$

Es brutto: $a_n = 2a_{n-1} + 2^n \qquad b_n = a_n + c n \cdot 2^n$

Fibonacci: $a_n = a_{n-1} + a_{n-2} \quad , \quad a_0 = 1, \quad a_1 = 1$

Es: $a_n = \boxed{3a_{n-1}} \boxed{-2a_{n-2}} \qquad a_0 = 1, \quad \boxed{a_1 = 1}$

$$a_n = c^n \qquad c \neq 0$$

$c^n = 3c^{n-1} - 2c^{n-2}$ dividendo per c^{n-2}

$$c^n - 3c^{n-1} + 2c^{n-2} = 0 \longrightarrow c^2 - 3c + 2 = 0$$

$c = 1$
$c = 2$

$$a_n = \alpha(1)^n + \beta(2)^n \qquad \begin{matrix} a_0 = 1 \\ a_1 = 1 \end{matrix}$$

$$\begin{cases} \alpha + \beta = 1 \\ \alpha + 2\beta = 1 \end{cases} \longrightarrow \begin{matrix} \alpha = \dots \\ \beta = \dots \end{matrix}$$